AD-A108 511   ARMY MISSILE COMMAND REDSTONE ARSENAL AL GUIDANCE A--ETC F/G 12/1
SOFTWARE FEATURES APPLICABLE TO INERTIAL MEASURFMENT UNIT SELF---ETC(U)
DEC 80 H V WHITE, J C HUNG, W G CANTRELL
UNCLASSIFIED DRSMI/RG-81-11-TR SBI-AD-E950 182 NL

1 of 1

END
DATE
FILMED
1-82
DTIC

1.0 4.5 5.0 5.6 6.3 2.8 3.2 3.6 4.0 2.5 2.2 2.0

1.1 1.8

1.25 1.4 1.6

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

AD-E 950 182

(12) LEVEL III

AD A108511

TECHNICAL REPORT RG-81-11

SOFTWARE FEATURES APPLICABLE TO INERTIAL MEASUREMENT UNIT SELF-ALIGNMENT

H. V. White
J. C. Hung
W. G. Cantrell
Guidance and Control Directorate
US Army Missile Laboratory

December 1980

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35809

Approved for public release; distribution unlimited.

81 12 09 059

DISPOSITION INSTRUCTIONS

DESTROY THIS REPORT WHEN IT IS NO LONGER NEEDED. DO NOT RETURN IT TO THE ORIGINATOR.

DISCLAIMER

THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.

TRADE NAMES

USE OF TRADE NAMES OR MANUFACTURERS IN THIS REPORT DOES NOT CONSTITUTE AN OFFICIAL INDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL HARDWARE OR SOFTWARE.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RG-81-11	2. GOVT ACCESSION NO. AD-A108 511	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Software Features Applicable to Inertial Measurement Unit Self Alignment		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H. V. White, J. C. Hung, W. G. Cantrell		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Commander, US Army Missile Command ATTN: DRSMI-RG Redstone Arsenal, AL 35898		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Commander, US Army Missile Command ATTN: DRSMI-RPT Redstone Arsenal, AL 35898		12. REPORT DATE December 1980 13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Approved for public release; distribution unlimited.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inertial measurement unit
Data reduction
Analytic Model
Scraping Technique
Continuous time regression algorithm

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report documents two software features applicable to inertial measurement unit self-alignment. Features are the continuous-time algorithm and the "scraping" technique.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

CONTENTS

I. INTRODUCTION

In the past several years the US Army Missile Command (MICOM) has been engaged in the development of techniques for improving the self-alignment performance of an inertial measurement unit (IMU) [1 to 12]. This report documents the continued findings of new techniques applicable to IMU self-alignment.

The precision of and the required time for IMU self-alignment depend not only on the quality of the IMU hardware but also on the effectiveness of the attendant software. In fact, an ingeniously developed self-alignment algorithm enables the compensation of anomalous effects caused by hardware imperfections. Thus, the stability of the hardware characteristics has become more important than the exactness of the hardware.

In general, an IMU self-alignment procedure contains on-line determination of a set of parameters characterizing the state of the IMU. To cope with the ever existing effect of random noise and disturbances, redundant measurements and a regression type of data reduction technique are used for improving parameter determination accuracy. The purpose of this report is to describe two features of data reduction for the self-alignment of a gimbaled IMU. The procedure involves an accelerometer based two-position self-alignment technique which has been reported in detail elsewhere [1, 5, 7]. The two features to be discussed are a continuous-time algorithm and a "scraping" technique. Since the present concern is the data reduction software, only the necessary background will be reviewed.

II. THE ANALYTIC MODEL

The parameters to be determined during IMU self-alignment are contained in the following two measurement equations,

$$V_N(t) = A_1 t + A_2 t^2 + \mu A_4 t^3 \qquad (1)$$

$$V_E(t) = A_3 t + A_4 t^2 - \mu A_2 t^3 \qquad (2)$$

where A_i, i=1 to 4, are the parameters, t is the time variable, μ is a constant, and $V_N(t)$ and $V_E(t)$ are observed north and east velocities. By letting t=kT, the discrete-time counterparts of Equations (1) and (2) are obtained as

$$V_N(kT) = B_1 k + B_2 k^2 + \nu B_4 k^3 \qquad (3)$$

$$V_E(kT) = B_3 k + B_4 k^2 - \nu B_2 k^3 \qquad (4)$$

where

$$B_1 = A_1 T \qquad B_2 = A_2 T^2$$

$$B_3 = A_3 T \qquad B_4 = A_4 T^2$$

$$v = \mu T.$$

The discrete-time model, given by Equations (3) and (4), was used by the present authors in the previous development of a data reduction algorithm.

III. CONTINUOUS-TIME REGRESSION ALGORITHM

The continuous-time regression algorithm differs from the discrete-time one in that the former uses integrations while the latter uses summations. It is apparent that the latter is the approximation for the former. A simple demonstration of this effect is given below.

Example: Consider a polynomial model

$$V(t) = at + bt^2 + ct^3. \tag{5}$$

and the associated integral square error

$$I_1 = \int_0^T \left[V(t) - (at + bt^2 + ct^3)\right]^2 dt. \tag{6}$$

The discrete-time versions of Equations (5) and (6) are

$$V(k) = a\tau k + b\tau^2 k^2 + c\tau^3 k^3 \tag{7}$$

and

$$I_2 = \sum_{k=1}^{N} \tau\left[V(k) - (a\tau k + b\tau^2 k^2 + c\tau^3 k^3)\right]^2 \tag{8}$$

where $N = T/\tau$. Elements of the matrices of the associated normal equations are

$$C_m = \int_0^T t^m dt, \qquad m = 2 \text{ to } 6 \tag{9}$$

for the continuous-time model, and

$$C_m = \tau^{m+1} \sum_{k=1}^{N} k^m, \qquad m = 2 \text{ to } 6 \tag{10}$$

for the discrete-time model. Let t=240 and τ=.192, then N=240/.192=1250. The computed values of C_m and C_m' are listed in Table I. Percent errors in C_m caused by use of the discrete-time model are also listed in the table.

Return to the IMU measurement model represented by Equations (1) and (2). The integral-square error resulting from a continuous-time least-square regression is given by

$$I(t_k) = \int_0^{t_k}\left[V_N(t) - (A_1 t + A_2 t^2 + \mu A_4 t^3)\right]^2 dt$$

$$+ \int_0^{t_k}\left[V_E(t) - (A_3 t + A_4 t^2 - \mu A_2 t^3)\right]^2 dt. \qquad (11)$$

Letting $\frac{\delta I}{\delta A_1} = \frac{\delta I}{\delta A_2} = \frac{\delta I}{\delta A_3} = \frac{\delta I}{\delta A_4} = 0$

yields the following set of normal equations which specify the condition for $I(t_k)$ to be minimum, where all integrations are from t=0 to $t=t_k$.

$$\int tV_N dt = A_1\int t^2 dt + A_2\int t^3 dt + \mu A_4\int t^4 dt. \qquad (12)$$

$$\int (t^2 V_N - \mu t^3 V_E)dt = A_1\int t^3 dt$$
$$+A_2\left(\int t^4 dt + \mu^2\int t^6 dt\right) - A_3\mu\int t^4 dt. \qquad (13)$$

$$\int tV_E dt = A_3\int t^2 dt + A_4\int t^3 dt - \mu A_2\int t^4 dt. \qquad (14)$$

$$\int (t^2 V_E + \mu t^3 V_N)dt = A_3\int t^3 dt$$
$$+ A_4\left(\int t^4 dt + \mu^2\int t^6 dt\right) + A_1\mu\int t^4 dt. \qquad (15)$$

Define

$$\begin{aligned} W_1 &= \int tV_N dt \\ W_2 &= \int (t^2 V_N - \mu t^3 V_E)dt \\ W_3 &= \int tV_E dt \\ W_4 &= \int (t^2 V_E + \mu t^3 V_N)dt \end{aligned} \qquad (16)$$

After evaluating the right-hand side integrals of Equations (12), (13), (14), and (15), this set of equations can be written in the matrix form as

TABLE I. VALUES OF ELEMENTS OF NORMAL EQUATION MATRICES

Continuous - Time Least - Square	Discrete - Time Least - Square	% Error In Discrete - Time Case
$C_2 = 4.608000 \times 10^6$	$C_2' = 4.613271 \times 10^6$	.11
$C_3 = 8.294400 \times 10^6$	$C_3' = 8.307429 \times 10^8$	.16
$C_4 = 1.592525 \times 10^{11}$	$C_4' = 1.595635 \times 10^{11}$	.20
$C_5 = 3.185050 \times 10^{13}$	$C_5' = 3.192629 \times 10^{13}$	.24
$C_6 = 6.552100 \times 10^{15}$	$C_6' = 6.570238 \times 10^{15}$	.28

$$\begin{bmatrix} W_1 \\ \\ W_2 \\ \\ W_3 \\ \\ W_4 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{t_k^3}{3} & \frac{t_k^4}{4} & 0 & \mu\frac{t_k^5}{5} \\ \\ \frac{t_k^4}{4} & \frac{t_k^5}{5} + \mu^2\frac{t_k^7}{7} & -\mu\frac{t_k^5}{5} & 0 \\ \\ 0 & -\mu\frac{t_k^5}{5} & \frac{t_k^3}{3} & \frac{t_k^4}{4} \\ \\ \mu\frac{t_k^5}{5} & 0 & \frac{t_k^4}{4} & \frac{t_k^5}{5} + \mu^2\frac{t_k^7}{7} \end{bmatrix}}_{G} \begin{bmatrix} A_1 \\ \\ A_2 \\ \\ A_3 \\ \\ A_4 \end{bmatrix} \qquad (17)$$

Let the 4x4 matrix in Equation (17) be denoted G and its elements g_{ij}. Thus

$$g_{11} = g_{33} = \frac{t_k^3}{3}$$

$$g_{12} = g_{21} = g_{34} = g_{43} = \frac{t_k^4}{4}$$

$$g_{22} = g_{44} = \frac{t_k^5}{5} + \mu^2\frac{t_k^7}{7} \qquad (18)$$

$$g_{14} = g_{41} = -g_{23} = -g_{32} = \mu\frac{t_k^5}{5}$$

$$g_{13} = g_{24} = g_{31} = g_{42} = 0$$

Let

$$H = G^{-1} \qquad (19)$$

and

$$A = g_{11}\, g_{22} - g_{12}^2 - g_{14}^2. \qquad (20)$$

Then, the elements of H, denoted h_{ij}, are given by

$$h_{11} = h_3 = \frac{g_{22}}{A} = K_1$$

$$h_{22} = h_{44} = \frac{g_{11}}{A} = K_2$$

$$h_{12} = h_{21} = h_{34} = h_{43} = -\frac{g_{12}}{A} = -K_3 \qquad (21)$$

$$h_{14} = h_{41} = -h_{23} = -h_{32} = \frac{-g_{14}}{A} = -K_4$$

$$h_{13} = h_{24} = h_{31} = h_{42} = 0$$

where K_i, i=1 to 4, is defined here to simplify the notation. The parameters A_i, i=1 to 4, are given by

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} K_1 & -K_3 & 0 & -K_4 \\ -K_3 & K_2 & K_4 & 0 \\ 0 & K_4 & K_1 & -K_3 \\ -K_4 & 0 & -K_3 & K_2 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} \qquad (22)$$

To evaluate K_i, use Equation (18) in (20) and (21) to give

$$K_1 = \frac{2400\ Q}{t_k^3 P}$$

$$K_2 = \frac{2800}{t_k^5 P}$$

$$K_3 = \frac{2100}{t_k^4 P} \qquad (23)$$

$$K_4 = \frac{1680\mu}{t_k^3 P}$$

where

$$P = 35 + 64\,\mu^2 t_k^2$$

$$Q = 7 + 5\,\mu^2 t_k^2 \qquad (24)$$

The evaluation of integrals of W_i, $i = 1$ to 4, as given by Equation (16) is more involved. First, let the velocity at any time be modeled by

$$V(t) = V(t_i) + a(t - t_i) \qquad (25)$$

where a is an estimated acceleration given by

$$a = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i} \qquad (26)$$

In Equations (25) and (26), t_i is the initial time and t_f the final time of a computation interval Δt. Also, $V_i = V(t_i)$ and $V_f = V(t_f)$. In the sequel, the notation $(\ )_k$ will be used to indicate that all quantities inside the parentheses are for the k-th computation interval. Next, consider W_1 of Equation (16).

$$(W_1)_k = (W_1)_{k-1} + \left(\int_{t_i}^{t_f} \left[V_{Ni}\,t + \frac{\Delta Vn}{\Delta t}\,(t - t_i)dt\right.\right)_k$$

$$= (W_1)_{k-1} + \left(V_{Ni}\,\frac{t_f^2 - t_i^2}{2}\right)_k$$

$$+ \left(\frac{V_{Nf} - V_{Ni}}{t_f - t_i}\right)_k \left(\frac{t_f^3 - t_i^3}{} - t_i\,\frac{t_f^2 - t_i^2}{2}\right)_k$$

After steps of algebraic simplification, one gets

$$(W_1)_k = (W_1)_{k-1} - \left(\frac{V_{Nf} - V_{Ni}}{6}\right)_k (t_f^2 + t_f t_i + t_i^2)_k$$

$$+ \left(\frac{V_{Nf} t_f^2 - V_{Ni} t_i^2}{2}\right)_k \tag{27}$$

Similarly, for W_3 of Equation (16),

$$(W_3)_k = (W_3)_{k-1} - \left(\frac{V_{Ef} - V_{Ei}}{6}\right)_k (t_f^2 + t_f t_i + t_i^2)_k$$

$$+ \left(\frac{V_{Ef} t_f^2 - V_{Ei} t_i^2}{2}\right)_k . \tag{28}$$

Finally, consider W_2 of Equation (16),

$$(W_2)_k = (W_2)_{k-1}$$

$$+ \left(\int_{t_i}^{t_f} \left[V_{Ni} + \frac{\Delta V_N}{\Delta t} (t - t_i) \right] t^2 dt \right.$$

$$\left. + \int_{t_i}^{t_f} \mu \left[V_{Ei} + \frac{\Delta V_E}{\Delta t} (t - t_i) \right] t^3 dt \right)_k$$

$$= (W_2)_{k-1} + \left(V_{Ni} \frac{t_f^3 - t_i^3}{3} \right)_k$$

$$+ \left(\frac{V_{Nf} - V_{Ni}}{t_f - t_i} \right)_k \left(\frac{t_f^4 - t_i^4}{4} - \frac{t_f^3 - t_i^3}{3} t_i \right)_k$$

After simplification,

$$(W_2)_k = (W_2)_{k-1} + \left(\frac{V_{Nf}t_f^3 - V_{Ni}t_i^3}{3}\right)_k - \mu\left(\frac{V_{Ef}t_f^4 - V_{Ei}t_i^4}{4}\right)_k$$

$$- \left(\frac{V_{Nf} - V_{Ni}}{12}\right)_k (t_f^3 + t_f^2\, t_i + t_f t_i^2 + t_i^3)_k$$

$$+ \mu\left(\frac{V_{Ef} - V_{Ei}}{20}\right)_k (t_f^4 + t_f^3 t_i + t_f^2 t_i^2 + t_f t_i^3 + t_i^4)_k. \qquad (29)$$

Similarly for W_4 of Equation (16),

$$(W_4)_k = (W_4)_{k-1} + \left(\frac{V_{Ef}t_f^3 - V_{Ei}t_i^3}{3}\right)_k + \mu\left(\frac{V_{Nf}t_f^4 - V_{Ni}t_i^4}{4}\right)_k$$

$$- \left(\frac{V_{Ef} - V_{Ei}}{12}\right)_k (t_f^3 + t_f^2 t_i + t_f t_i^2 + t_i^3)_k$$

$$- \mu\left(\frac{V_{Nf} - V_{Ni}}{20}\right)_k (t_f^4 + t_f^3 t_i + t_f^2 t_i^2 + t_f t_i^3 + t_i^4)_k. \qquad (30)$$

Equations (27) to (30) are recursive updating equations for W_i which are needed in Equation (22) for the recursive determination of A_i.

Equations (22), (23), (24), (27), (28), (29), and (30) constitute the set of continuous-time regression algorithms.

IV. THE SCRAPING FEATURE

Consider a regression model

$$V = Ak + Bk^2, \quad k = 0, 1, 2, \ldots \tag{31}$$

where V is measured while A and B are parameters to be determined. Assume that B is much smaller than A. Under this condition the determination of B will be less accurate than that of A because of computation round-off errors. One way to get around this difficulty is to subtract from V the amount $\hat{A}k$ where $\hat{A}$ is the a priori value of A. $\hat{A}$ is related to A by

$$A = \hat{A} + \Delta A \tag{32}$$

where ΔA is the error in $\hat{A}$. Thus, a new regression model is given by

$$V' = V - \hat{A}k = \Delta Ak + Bk^2. \tag{33}$$

Usually ΔA is small enough not to exert a dominating effect on B. For a recursive regression algorithm, the part of Ak known a priori can "scraped" off from V recursively. Therefore, this feature will be called the "scraping feature". A demonstration of the effectiveness of this feature is given below.

Example: Consider measurements generated by the model

$$V_k = Ak + Bk^2 + n_k, \quad k = 1 \text{ to } 100$$

where A = 1.111111 and B = .000001. Measurement noise n_k has a normal distribution of mean zero and variance one. Least square regression with and without the scraping feature are used to estimate A and B. Results are shown in Table II. Improved results due to the scraping feature are evident.

Return to the IMU measurement model of Equations (1) and (2). Here, accurate determination of parameters A_2 and A_4 is more important than that of A_1 and A_3. The scraping feature will be incorporated into the regression algorithm to enhance the accuracy of the estimated A_2 and A_4. Define

$$\left.\begin{aligned} V_N' &= V_N - \hat{A}_1 t = \Delta A_1 t + A_2 t^2 + \mu A_4 t^3 \\ V_E' &= V_E - \hat{A}_3 t = \Delta A_3 t + A_4 t^2 - \mu A_2 t^3 \end{aligned}\right\} \quad . \tag{34}$$

TABLE II. LEAST - SQUARE REGRESSIONS WITH and WITHOUT SCRAPING FEATURE

$$A = 1.11111$$

$$B = .000001$$

Least - square regression with scraping feature

K	ESTIMATES			
	$\hat{A}$	% Error		% Error
10	1.1111	0.0000	9.45809×10^{-7}	-5.4
50	1.11111	0.0000	1.00060×10^{-6}	0.06
100	1.11111	0.0000	9.99795×10^{-7}	0.02

Least - square regression without scraping feature

K	ESTIMATES			
	$\hat{A}$	% Error	$\hat{B}$	% Error
10	1.11111	0.0000	9.53674×10^{-7}	-4.6
50	1.11112	0.0007	8.34465×10^{-7}	-16.6
100	1.11113	0.0014	8.34465×10^{-7}	-16.6

where

$$\left.\begin{aligned} A_1 &= \hat{A}_1 + \Delta A_1 \\ A_3 &= \hat{A}_3 + \Delta A_3 \end{aligned}\right\} \qquad (35)$$

Equation (34) is the new regression model for IMU self-alignment. Comparing Equation (34) to (1) and (2), one notes the following replacements:

ΔA_1 replaces A_1

ΔA_3 replaces A_3

V_N' replaces V_N

V_E' replaces V_E.

Therefore, using similar replacements, the least-square results obtained in Section 3 can be modified for the model of Equation (34). From Equation (22), with proper replacement, one gets

$$\begin{bmatrix} \Delta A_1 \\ A_2 \\ \Delta A_3 \\ A_4 \end{bmatrix}_k = \begin{bmatrix} K_1 & -K_3 & 0 & -K_4 \\ -K_3 & K_2 & K_4 & 0 \\ 0 & K_4 & K_1 & -K_3 \\ -K_4 & 0 & -K_3 & K_2 \end{bmatrix}_k \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix}_k \qquad (36)$$

where K_i are given by Equations (23) and (24), and W_i are obtained from Equation (16) as

$$W_1 = \int tV_N' dt$$

$$\left.\begin{aligned} W_2 &= \int (t^2 V_N' - t^3 V_E')dt \\ W_3 &= \int t V_E' dt \\ W_4 &= \int (t^2 V_E' + t^3 V_N')dt \end{aligned}\right\} \quad (37)$$

Once $(\Delta A_1)_k$ and $(\Delta A_3)_k$ are determined, $(A_1)_k$ and $(A_3)_k$ are given by

$$\left.\begin{aligned} (A_1)_k &= (A_1)_{k-1} + (\Delta A_1)k \\ (A_3)_k &= (A_3)_{k-1} + (\Delta A_3)_k \end{aligned}\right\} \quad (38)$$

The algorithm for W_1 of Equation (37) is more involved than that given by Equations (27) to (30). First consider the scraping of V_N to give V_N'.

$$\begin{aligned} (V_N')_k &= (V_N)_k - (A_1)_{k-1} t_k \\ &= (V_N)_{k-1} + (\Delta V_N)_k \\ &\quad - \left[(A_1)_{k-2} + (\Delta A_1)_{k-1}\right] (t_{k-1} + \Delta t) \\ &= \underbrace{(V_N)_{k-1} + (A_1)_{k-2} t_{k-1}}_{(V_N')_{k-1}.} + (\Delta V_N)_k - (A_1)_{k-2} \Delta t \\ &\quad - (\Delta A_1)_{k-1} t_k \end{aligned}$$

$(V_E')_k$ can be obtained from $(V_E)_k$ in a similar way. Hence

$$\left.\begin{aligned} (V_N')_k &= (V_N')_{k-1} + (\Delta V_N)_k - (A_1)_{k-2} \Delta t - (\Delta A_1)_{k-1} t_k \\ (V_E')_k &= (V_E')_{k-1} + (\Delta V_E)_k - (A_3)_{k-2} \Delta t - (\Delta A_3)_{k-1} t_k. \end{aligned}\right\} \quad (39)$$

Next, consider the effect of ΔA_1, and ΔA_3 on W_i. From Equation (17), replacing A_1 and A_3 by ΔA_1 and ΔA_3, respectively,

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix}_k = \begin{bmatrix} \frac{t_k^3}{3} & \frac{t_k^4}{4} & 0 & \frac{\mu t_k^5}{5} \\ \frac{t_k^4}{4} & \frac{t_k^5}{5} + \frac{\mu^2 t_k^7}{7} & \frac{-\mu t_k^5}{5} & 0 \\ 0 & \frac{-\mu t_k^5}{5} & \frac{t_k^3}{3} & \frac{t_k^4}{4} \\ \frac{\mu t_k^5}{5} & 0 & \frac{t_k^4}{4} & \frac{t_k^5}{5} + \frac{\mu^2 t_k^7}{7} \end{bmatrix}_k \begin{bmatrix} \Delta A_1 \\ A_2 \\ \Delta A_3 \\ A_4 \end{bmatrix}_k \quad (40)$$

The part of W_i due to ΔA_1 and ΔA_3 alone are, therefore, given by

$$(\Delta W_1)_k = \frac{t_k^3}{3} (\Delta A_1)_k$$

$$(\Delta W_2)_k = \frac{t_k^4}{4} (\Delta A_1)_k - \frac{\mu t_k^5}{5} (\Delta A_3)_k$$

$$(\Delta W_3)_k = \frac{t_k^3}{3} (\Delta A_3)_k$$

$$(\Delta W_4)_k = \frac{t_k^4}{4} (\Delta A_3)_k + \frac{\mu t_k^5}{5} (\Delta A_1)_k$$

$(\Delta W_i)_k$ are the a posteriori correction for $(W_i)_k$ after $(\Delta A_i)_k$, $(\Delta A_1)_k$, and $(\Delta A_3)_k$ are estimated. Ideally $\Delta A_1 = \Delta A_3 = 0$, therefore, the corrected $(W_i)_k$ *denoted* $(W_i^c)_k$ *are given by*

$$(W_i^c)_k = (W_i)_k - (\Delta W_i)_k, \quad i = 1 \text{ to } 4. \tag{42}$$

Now one is ready to modify Equations (27) to (30) for the algorithm for W_i. The modifications consist of replacing:

V_{Nf} by V'_{Nf}

V_{Ni} by V'_{Ni}

V_{Ef} by V'_{Ef}

V_{Ei} by V'_{Ei}.

$(W_i)_{k-1}$ at the right-hand side by $(W_i^c)_{k-1}$

The result is

$$\begin{aligned}(W_1)_k = {} & (W_1)_{k-1} - (\Delta W_1)_{k-1} \\ & - \frac{1}{6}(V'_{nf} - V'_{Ni})_k (t_f^2 + t_f t_i + t_i^2)_k \\ & + \frac{1}{2}(V'_{Nf} t_f^2 - V'_{Ni} t_i^2)_k.\end{aligned} \tag{43}$$

$$\begin{aligned}(W_2)_k = {} & (W_2)_{k-1} - (\Delta W_2)_{k-1} \\ & - \frac{1}{12}(V'_{nf} - V'_{Ni})_k (t_f^3 + t_f^2 t_i + t_f t_i^2 + t_i^2)_k \\ & - \frac{1}{3}(V'_{Nf} t_f^3 - V'_{Ni} t_i^3)_k \\ & + \frac{\mu}{20}(V'_{Ef} - V'_{Ei})_k (t_f^4 + t_f^3 t_i + t_f^2 t_i^2 + t_f t_i^3 + t_f^4)_k \\ & - \frac{\mu}{4}(V'_{Ef} t_f^4 - V'_{Ei} t_i^4)_k\end{aligned} \tag{44}$$

$$(W_3)_k = (W_3)_{k-1} - (\Delta W_3)_{k-1}$$

$$- \frac{1}{6} (V'_{Ef} - V_{Ei})_k \, (t_f^2 + t_f t_i + t_i^2)_k$$

$$+ \frac{1}{2} (V'_{Ef} t_f - V_{Ei} t_i)_k . \qquad (45)$$

$$(W_4)_k = (W_4)_{k-1} - (\Delta W_4)_{k-1}$$

$$- \frac{1}{12} (V'_{Ef} - V'_{Ei})_k \, (t_f^3 + t_f^2 t_i + t_f t_i^2 + t_i^3)_k$$

$$+ \frac{1}{3} (V'_{Ef} t_f^3 - V_{Ei} t_i^3)_k$$

$$- \frac{\mu}{20} (V'_{Nf} - V'_{Ni})_k \, (t_f^4 + t_f^3 t_i + t_f^2 t_i^2 + t_f t_i^3 + t_i^4)_k$$

$$+ \frac{\mu}{4} (V'_{Nf} t_f^4 - V'_{Ni} t_i^4)_k . \qquad (46)$$

The complete set of least-square regression algorithms with the scraping feature is given by Equations (36), (38), (39), (41), (43), (44), (45), and (46).

Testing the Scraping Feature

A least-square regression algorithm with scraping feature was implemented into an IMU self-alignment and gyrocompassing software. The software was then tested on a long range missile IMU. A total of 1223 sets of incremental velocities ΔV_N, ΔV_E, and ΔV_A was measured. The algorithm was recursive which yielded 1222 sets of alignment results beginning with the second measurement set. For the sake of comparison, another test run using a least-square algorithm without the scraping feature was also made.

It was predicted that the algorithm with scraping would result in faster convergence as compared to the algorithm without scraping. This pre-

diction was confirmed by the test results, although the improvement is small. Figure 1 shows the convergence plot of values for α, the heading angle, D_N', the north channel drift, and D_E', the east channel drift.

Algorithms used in the test are listed in the appendices. Appendix A contains the algorithm with the scraping feature, and Appendix B contains the one without the feature.

V. CONCLUDING REMARKS

Two software features have been described in this report. The features are applicable to IMU self alignment and gyrocompassing. The first feature is the use of continuous-time arithmetic in a least-square regression algorithm. This feature avoids the error caused by the discrete-time model. The second feature, called scraping, is intended to enhance the accuracy of determining the drift parameters by recursively scraping off from measured velocities those components caused by platform tilt misalignment. The result of scraping is to reduce the effect of computer round-off error. In the sample test, the improvement of gyrocompassing by scraping is small. This is due to the small round-off error to begin with. In the case where round-off error is severe, scraping should provide appreciable improvement in gyrocompassing.

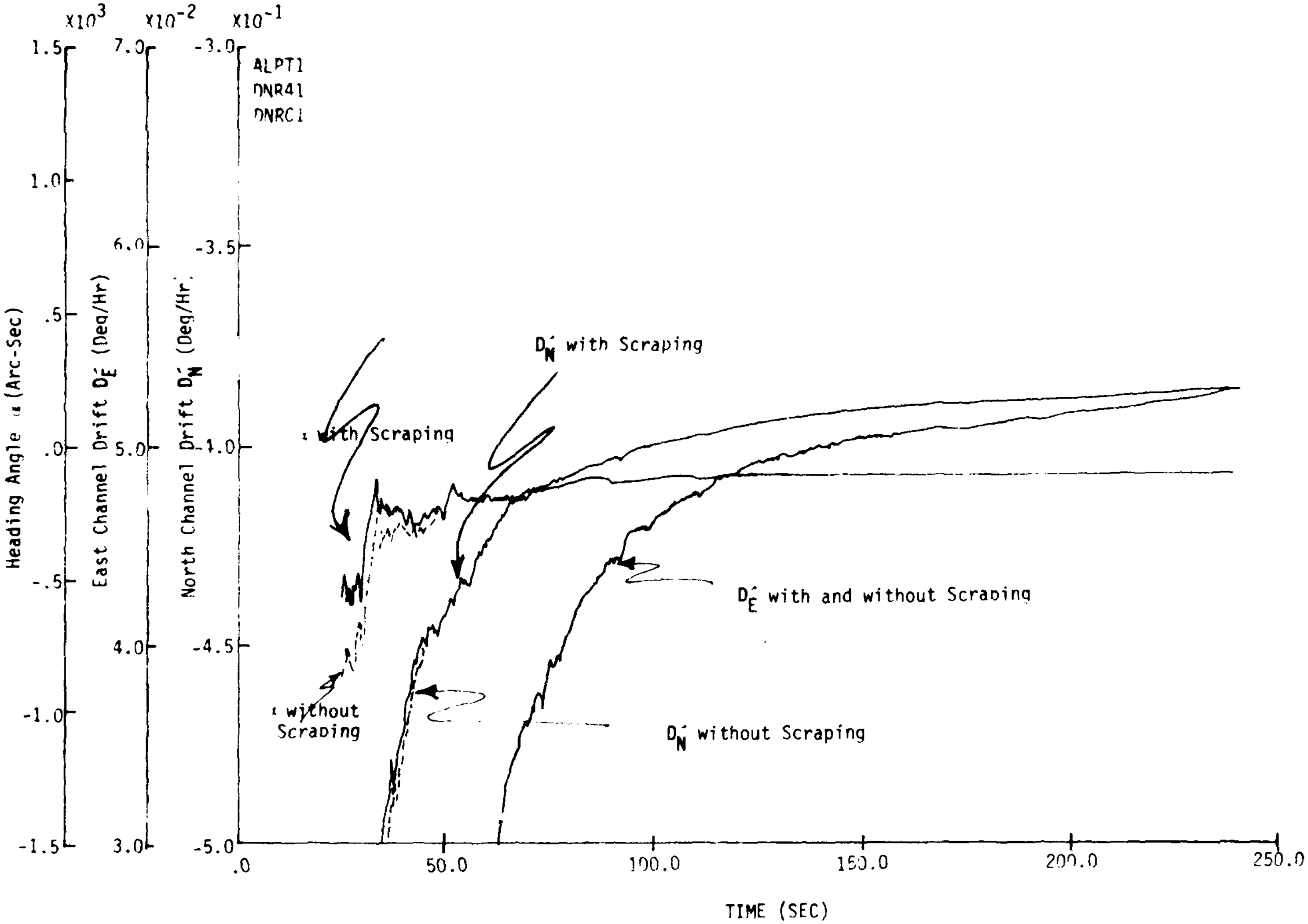

Figure I. Convergence of α, $\hat{D}_N$, and $\hat{D}_E$.

APPENDIX A. FORTRAN PROGRAM FOR LEAST - SQUARE WITH SCRAPING

```
FIN4.L
      SUBROUTINE AL13(INF)
C
C*******************************************
C
C       STORED AS "&AL13X" ON DISC
C
C*******************************************
C
C---NEW MICOM LEAST SQUARE FIT ROUTINE WITH SCRAPPING FEATURE
C               29 SEPT 80
C
C      THIS ROUTINE COMPUTES ESTIMATES OF TILT AND TILT RATE
C      BY PERFORMING A LEAST SQUARES FIT OF VELOCITY DATA IN
C      REFERENCE COORDINATES
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      STANDARD COMMON
C
C      7/30/74
C
C
      REAL K1S,K0,K1,K2,K3,KS,KT,KNOM,KTX,LAMDA,LAT,IDENT,
     $  MTRXA,MTRXB,MTRXC,KTA,KTD,MODIP
C
      INTEGER GYPSL,AZGYP,SLEW,ODISC,ZSFLG,HLFLG,CCFLG,ERFG,SFLAG,
     $  TEM2,AFLAG,FST,YEAR,DAY,HOURS,SECS,ERRCT,AXIS
     $  ,RTEIO,SYNC,ERROR
C
      COMMON ANGLE(3),SMANG(6),K1S(2,3,2),THET(3),GA(3),DTHET,GYPSL,
     B  AZGYP,SLEW(6),IMU01,IMU02,IMUIN(16),IDISC,ODISC,DELC(3),
     C  NUM(6),RDCL(3),ZSFLG,HLFLG,CCFLG,MDFLG,IRFLG,IFFLG,ERFG(3),
     D  NEGCT,THEA(3),K0(2,3),K1(2,3),K2(3),K3(3),KS(2,3),KT(3),
     E  KNOM(2),KTX(3),DELTA(6),DV(3),TORQ(3),IDUM1,JUP,SA(3),ETDU,
     F  CGD,WE(3),ER(3),DELTT,RDELT,CNT,RCNT,TRP(3,3),RMAX,
     G  RMAXH,LAMDA,LAT(3),DF(3),DI(3),DO(3),DS(3),G,BETA(6),GT(3),
     H  TMINR,THLT1,THET2,DVR(3),ETD(3),IDUM2,SFLAG,OMEGE,TRC(3,3),
     J  TRD(3,3),TPD(3,3),PHIZ,PHIC,DEGRA,CONVF,IDUM3,AFLAG,FST(12),
     K  YEAR,MONTH,DAY,HOURS,MINS,SECS,MSECS,COLD(2,3),DCOLD(2,3),
     L  CNEW(2,3),DCNEW(2,3),C(2,3),DC(2,3),ERRCT,IPF,MODIP,UTYIP,TEM2,
     M  IDENT(3,3),MTRXA(3,3),MTRXB(3,3),MTRXC(3,3),THD,THF,TCP(3,3),
     N  TCD(3,3),TDR,KTA(3),KTD(3),TFIUP(3,3),THETA(6),AXIS(3),
     P  TLSF,TLAT,FR(3),THETM(5),ETD1(2),
     Q TIME(250),THNOR(250),THWES(250),ETDNO(250),ETDWS(250),KK,TH3(250)
     R  ,ALPTP(125),KPLOT,X1P(250),X2P(250),X3P(250),X4P(250),X5P(250)
     S  ,DNRCP(125),DNRCQ(125),NSEQ7,ALPTQ(125)
     T  ,RTEIO,SYNC,NBUF,IMUSN,ISPACE,BUF1(30),BUF2(32),ERROR
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DIMENSION INLSF(2)
      EQUIVALENCE (THETM(1),X1),(THETM(2),X2),(THETM(3),X3),
     A  (THETM(4),X4),(THETM(5),X5),(INLSF,TLSF),(INLSF(2),K)
      EQUIVALENCE (FR(3),FRU)
```

```
      DATA I13/167777B/
C     DETERMINE IF AL13 IS TO BE EXECUTED
C
      IF(INF.NE.0) GO TO 1
      FST10=0
      RETURN
C
C     DETERMINE IF FIRST OR SECOND TIME THROUGH.  FIRST TIME
C     THROUGH WILL NOT COMPUTE THE LSF
C
    1 IF(FST10-1)2,4,4
C
C     FIRST TIME THROUGH SET NUMBER OF ITERATIONS
C     AND INITIAL CONDITIONS
C
    2 FST10=1
      NLSF=INLSF
C
C
C     CONVERT FROM DEG/HR TO RAD/SEC
C
      FFRU=FRU/(3600.*DEGRA)
      U=-FFRU/3.
C
      T1=0.
      DTSQ=DELTT*DELTT
      DTCB=DTSQ*DELTT
      DTQD=DTSQ*DTSQ
      A1=.3333333333*DTSQ
      A2=.25*DTCB
      A3=.2*DTQD*U
C
      VNF=DVR(1)
      VEF=-DVR(2)
      VAF=-DVR(3)
C
      W1=VNF*A1
      W2A=VNF*A2
      W2B=-VEF*A3
      W3=VEF*A1
      W4A=VEF*A2
      W4B=VNF*A3
      W5=VAF*A1
      K=1
      DX1=0.
      DX3=0.
      X1 =0.
      X11 = 0.0
      X3 =0.
      X33 = 0.0
      GO TO 5
C
C     SECOND AND SUBSEQUENT TIMES THROUGH, COMPUTE
C       W1 TO W5, S1 TO S5, AND  X1 TO X5
C
    4 K=K+1
```

```
C
C      COMPUTE W1 TO W5
C
       TI=TI+DELTT
       TF=TI+DELTT
C
       VNI=VNF
       VNF=VNI+DVR(1) - X11*DELTT - DX1*TF
       VEI=VEF
       VEF=VEI+DVR(2) - X33*DELTT - DX3*TF
       VAI=VAF
       VAF=VAI+DVR(3)
C
       TFSQ=TF*TF
       TFCB=TFSQ*TF
       TFQD=TFSQ*TFSQ
       TFQT=TFSQ*TFCB
       TISQ=TI*TI
       TICB=TISQ*TI
       TIQD=TISQ*TISQ
C
       T2ND=TFSQ+TF*TI+TISQ
       T3RD=TFCB+TFSQ*TI+TF*TISQ+TICB
       T4TH=TFQD+TFCB*TI+TFSQ*TISQ+TF*TICB+TIQD
C
       A1=.3333333333*TFCB*DX1
       A2=.25*TFQD*DX1-.2*U*TFQT*DX3
       A3=.3333333333*TFCB*DX3
       A4=.25*TFQD*DX3+.2*U*TFQT*DX1
C
       W1=W1-(VNF-VNI)*T2ND/6.+(VNF*TFSQ-VNI*TISQ)*.5-A1
       W2A=W2A-(VNF-VNI)*T3RD/12.+(VNF*TFCB-VNI*TICB)/3.
       W2B=W2B+U*((VEF-VEI)*T4TH*.05-(VEF*TFQD-VEI*TIQD)*.25)-A2
       W3=W3-(VEF-VEI)*T2ND/6.+(VEF*TFSQ-VEI*TISQ)*.5-A3
       W4A=W4A-(VEF-VEI)*T3RD/12.+(VEF*TFCB-VEI*TICB)/3.
       W4B=W4B-U*((VNF-VNI)*T4TH*.05-(VNF*TFQD-VNI*TIQD)*.25)-A4
       W5=W5-(VAF-VAI)*T2ND/6.+(VAF*TFSQ-VAI*TISQ)*.5
C
C
C      COMPUTE S1 TO S5
C
C
       TFU=TF*U
       TFUSQ=TFU*TFU
       PP=35.+64.*TFUSQ
       QQ=7.+5.* TFUSQ
       RRRB=TFCB*PP
       RRRC=RRRB*TF
       RRRD=RRRC*TF
C
       S1=240.*QQ/RRRB
       S2=2800./RRRD
       S3=2100./RRRC
       S4=1680.*U/RRRB
       S5= 3./TFCB
C
```

```
C      COMPUTE X1 TO X5
C
       X1A=S1*W1-S3*W2A-S4*W4A
       X1B=       -S3*W2B-S4*W4B
       DX1=X1A+X1B
       X11  = X1
       X1  =X1+DX1
       X2A=-S3*W1+S2*W2A+S4*W3
       X2B=          S2*W2B
       X2=X2A+X2B
       X3A= S4*W2A+S1*W3-S3*W4A
       X3B= S4*W2B          -S3*W4B
       DX3=X3A+X3B
       X33 = X3
       X3  =X3+DX3
       X4A=-S4*W1-S3*W3+S2*W4A
       X4B=                S2*W4B
       X4=X4A+X4B
       X5= S5*W5
C
C     FILL BUFFER
C
     5 KTE10 = 125B
       BUF1(6) = 13
       DO 10 I = 1,3
        BUF1(I+23) = ER(I)
        BUF1(I+20) = ETD(I)
   10   BUF1(I+6) = DVR(I)
       DO 20 I = 1,6
   20   BUF1(I+9) = THETA(I)
       DO 30 I = 1,5
   30   BUF1(I+15) = THETM(I)
       BUF1(27) = TLSF
C
       IF(K.LE.NLSF)RETURN
C
       AFLAG=IAND(I13,AFLAG)
       FST10=0
       END
```

APPENDIX B. FORTRAN PROGRAM FOR LEAST-SQUARE WITHOUT SCRAPING

```
FTN4
      SUBROUTINE AL13(INC)
C
C*******************************************************
C
C        STORED AS "&AL13B" ON DISC
C              REV. 25 SEPT 80
C
C*******************************************************
C
C       NEW MICOM LEAST SQUARE FIT ROUTINE
C        7AUG78    REV. 13AUG80 1
C
C       THIS ROUTINE COMPUTES ESTIMATES OF TILT AND TILT RATE
C       BY PERFORMING A LEAST SQUARES FIT OF VELOCITY DATA IN
C       REFERENCE COORDINATES
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C       STANDARD COMMON
C
C       7/30/74
C
C
       REAL K1S,K0,K1,K2,K3,K5,KT,KNOM,KTX,LAMDA,LAT,IDENT,
      $  MIRXA,MIRXB,MIRXC,KTA,KTD,MODIP
C
       INTEGER GYPSL,AZGYP,SLEW,ODISC,ZSFLG,HFFLG,CCFLG,ERFG,SFLAG,
      $  TEM2,AFLAG,FST,YEAR,DAY,HOURS,SECS,ERRCT,AXIS
      $  ,RTEIO,SYNC,ERROR
C
       COMMON ANGLE(3),SMANG(6),K1S(2,3,2),THET(3),GA(3),DTHET,GYPSL,
      B  AZGYP,SLEW(5),IMU01,IMU02,IMUIN(16),IDISC,ODISC,DELC(3),
      C  NUM(6),RDCL(3),ZSFLG,HFFLG,CCFLG,MDFLG,IRFLG,IFFLG,ERFG(3),
      D  NEGCT,THEA(3),K0(2,3),K1(2,3),K2(3),K3(3),K5(2,3),KT(3),
      E  KNOM(2),KTX(3),DELTA(6),DV(3),TORQ(3),IDUM1,JUP,SA(3),ETDU,
      F  CGD,WE(3),ER(3),DELTT,RDELT,CNT,RCNT,TRP(3,3),RMAX,
      G  RMAXH,LAMDA,LAT(3),DF(3),DI(3),DU(3),DS(3),G,BETA(6),GT(3),
      H  TMINR,THET1,THET2,DVR(3),ETD(3),IDUM2,SFLAG,OMEGE,TRC(3,3),
      J  TRD(3,3),TPD(3,3),PHI2,PHIC,DEGRA,CONVF,IDUM3,AFLAG,FST(12),
      K  YEAR,MONTH,DAY,HOURS,MINS,SECS,MSECS,COLD(2,3),DCOLD(2,3),
      L  CNEW(2,3),DCNEW(2,3),C(2,3),DC(2,3),ERRCT,TPF,MODIP,UTYIP,TEM2,
      M  IDENT(3,3),MIRXA(3,3),MIRXB(3,3),MIRXC(3,3),THD,THF,TCP(3,3),
      N  TCD(3,3),TDR,KTA(3),KTD(3),TF1UP(3,3),THETA(6),AXIS(3),
      P  TLSF,TLAT,FR(3),THETM(5),ETDT(2),
      QTIME(250),THNOR(250),THWLS(250),ETDNO(250),ETDWS(250),KK,THS(250)
      R  ,ALPTP(125),KPLOT,X1P(250),X2P(250),X3P(250),X4P(250),X5P(250)
      S  ,DNRCP(125),DNRCQ(125),NSLOZ,ALPTQ(125)
      T  ,RTEIO,SYNC,NBUF,IMUSN,ISPACE,BUF1(30),BUF2(32),ERROR
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
       DIMENSION INLSF(2)
       EQUIVALENCE (THETM(1),X1),(THETM(2),X2),(THETM(3),X3),
      A  (THETM(4),X4),(THETM(5),X5),(INLSF,TLSF),(INLSF(2),K)
```

```
      EQUIVALENCE (FR(3),FRU)
      DATA I13/167777B/
C     DETERMINE IF AL13 IS TO BE EXECUTED
C
      IF(INI.NE.0) GO TO 1
      FST10=0
      RETURN
C
C     DETERMINE IF FIRST OR SECOND TIME THROUGH.  FIRST TIME
C     THROUGH WILL NOT COMPUTE THE LSF
C
    1 IF(FST10-1)2,4,4
C
C     FIRST TIME THROUGH SET NUMBER OF ITERATIONS
C     AND INITIAL CONDITIONS
C
    2 FST10=1
      NLSF=INLSF
C
C
C     CONVERT FROM DEG/HR TO RAD/SEC
C
      FFRU=FRU/(3600.*DEGRA)
C
C  MINUS SIGN INSERTED. MAY BE REMOVED WITH $ON,57
C
      U =  -FFRU/3.
C
      T1=0.
      DTSQ=DELTT*DELTT
      DTCB=DTSQ*DELTT
      DTQD=DTSQ*DTSQ
      A1=.3333333333*DTSQ
      A2=.25*DTCB
      A3=.2*DTQD*U
C
      VNF=DVR(1)
      VEF=-DVR(2)
      VAF=-DVR(3)
C
      W1=VNF*A1
      W2A=VNF*A2
      W2B=-VEF*A3
      W3=VEF*A1
      W4A=VEF*A2
      W4B=VNF*A3
      W5=VAF*A1
      K=1
      GO TO 5
C
C     SECOND AND SUBSEQUENT TIMES THROUGH, COMPUTE
C       W1 TO W5, S1 TO S5, AND  X1 TO X5
C
    4 K=K+1
C
C     COMPUTE W1 TO W5
```

```
C
      TI=TI+DELTT
      TF=TI+DELTT
C
      VNI=VNF
      VNF=VNI+DVR(1)
      VLI=VLF
      VEF=VEI+DVR(2)
      VAI=VAF
      VAF=VAI+DVR(3)
C
      TFSQ=TF*TF
      TFCB=TFSQ*TF
      TFQD=TFSQ*TFSQ
      TISQ=TI*TI
      TICB=TISQ*TI
      TIQD=TISQ*TISQ
C
      T2ND=TFSQ+TF*TI+TISQ
      T3RD=TFCB+TFSQ*TI+TF*TISQ+TICB
      T4TH=TFQD+TFCB*TI+TFSQ*TISQ+TF*TICB+TIQD
C
      W1=W1-(VNF-VNI)*T2ND/6.+(VNF*TFSQ-VNI*TISQ)*.5
      W2A=W2A-(VNF-VNI)*T3RD/12.+(VNF*TFCB-VNI*TICB)/3.
      W2B=W2B+U*((VLF-VLI)*T4TH*.05-(VLF*TFQD-VLI*TIQD)*.25)
      W3=W3-(VEF-VEI)*T2ND/6.+(VEF*TFSQ-VEI*TISQ)*.5
      W4A=W4A-(VEF-VEI)*T3RD/12.+(VEF*TFCB-VEI*TICB)/3.
      W4B=W4B-U*((VNF-VNI)*T4TH*.05-(VNF*TFQD-VNI*TIQD)*.25)
      W5=W5-(VAF-VAI)*T2ND/6.+(VAF*TFSQ-VAI*TISQ)*.5
C
C
C     COMPUTE S1 TO S5
C
C
      TFU=TF*U
      TFUSQ=TFU*TFU
      PP=35.+64.*TFUSQ
      QQ=7.+5.*TFUSQ
      RRRB=TFCB*PP
      RRRC=RRRB*TF
      RRRD=RRRC*TF
C
      S1=240.*QQ/RRRB
      S2=2800./RRRD
      S3=2100./RRRC
      S4=1680.*U/RRRB
      S5=3./TFCB
C
C     COMPUTE X1 TO X5
C
      X1A=S1*W1-S3*W2A-S4*W4A
      X1B=        S3*W2B-S4*W4B
      X1=X1A+X1B
      X2A=-S3*W1+S2*W2A+S4*W3
      X2B=        S2*W2B
      X2=X2A+X2B
```

```
      X3A= S4*W2A+S1*W3-S3*W4A
      X3B= S4*W2B         S3*W4B
      X3=X3A+X3B
      X4A=-S4*W1-S3*W3+S2*W4A
      X4B=                S2*W4B
      X4=X4A+X4B
      X5= S5*W5
C
C     FILL BUFFER
C
    5 KTE10 = 125B
      BUF1(6) = 13
      DO 10 I = 1,3
       BUF1(I+23) = ER(I)
       BUF1(I+20) = EID(I)
   10  BUF1(I+6) = DVR(I)
      DO 20 I = 1,6
   20  BUF1(I+9) = THETA(I)
      DO 30 I = 1,5
   30  BUF1(I+15) = THETM(I)
      BUF1(27) = TLSF
C
      IF(K.LE.NLSF)RETURN
C
      AFLAG=IAND(113,AFLAG)
      FST10=0
      END
```

REFERENCES

1. Hung, J. C. and White, H. V., IMU Self-Alignment Techniques, US Army Missile Command, Redstone Arsenal, Alabama, September 1974, Report No. RG-75-15.

2. Hung, J. C. and White, H. V., "Parameter Correlated Least Square Regression Algorithm," Proceedings of Asilomar Conference Circuits, Systems and Computers, Pacific Grove, California, 3-5 December 1974, pp. 415-419.

3. Hung, J. C. and White, H. V., "Error Analysis for Parameter Correlated Least Square Regression Algorithm," Proceedings of 1975 Southeastern Symposium on System Theory, Auburn and Tuskegee, Alabama, 20-21 March 1975.

4. White, H. V., Description of Two-Axis Table Method for PERSHING II Inertial Measurement Unit Gyrocompassing Verification, US Army Missile Command, Redstone Arsenal, Alabama, June 1975, Report No. RG-75-57.

5. Hung, J. C. and White, H. V., "On IMU Self-Alignment Algorithms," Proceedings of the Sixth Triennial World Congress of the International Federation of Automatic Control, Instrument Society of America, Pittsburg, Pennsylvania, August 1975.

6. Hung, J. C. and White, H. V., More On IMU Self-Alignment Techniques, US Army Missile Command, Redstone Arsenal, Alabama, September 1975, Report No. RG-76-23.

7. Hung, J. C. and White, H. V., "Self-Alignment Techniques for Inertial Measurement Units," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-11, No. 6, November 1975.

8. Hung, J. C. and White, H. V., "An Efficient Inertial Measurement Unit (IMU) Alignment Software," Proceedings of the IEEE Position Location and Navigation Symposium (PLANS 76), IEEE Publication 76CH1138-7 AES, New York, New York, 1-3 November 1976, pp. 128-132.

9. White, H. V., Pershing PII Inertial Measurement Unit Field Gyrocompassing Test, US Army Missile Research and Development Command, Redstone Arsenal, Alabama, June 1977, Report No. TG-77-16.

10. White, H. V. and Hung, J. C., "Field Test and Data Analysis for PII IMU Gyrocompassing," Proceedings of the IEEE Position Location and Navigation Symposium (PLANS 78), San Diego, California, 7-9 November 1978.

11. White, H. V. and Hung, J. C., Gyrocompassing Error Analysis for PERSHING II Inertial Measurement Unit, US Army Missile Research and Development Command, Redstone Arsenal, Alabama, November 1978, Report No. T-79-1.

12. Hung, J. C. and White, H. V., "Identification of IMU Parameters by Gyrocompassing Test," Proceedings of the International Federation of Automatic Control Symposium on Estimation and System Parameter Identification, Pergamon Press, London, England, August 1979.

DISTRIBUTION

	No. Of Copies
Commander US Army Materiel Development and Readiness Command ATTN: DRCRD	1
DRCDL	1
5001 Eisenhower Avenue Alexandria, Virginia 22333	
The University of Tennessee Department of Electrical Engineering ATTN: Dr. J. C. Hung Knoxville, Tennessee 37916	1
NASA Johnson Space Center EG5 ATTN: Mr. Malcolm Jones Houston, Texas 77058	1
US Army Materiel Systems Analysis Activity ATTN: DRXSY-MP Aberdeen Proving Ground, Maryland 21005	1
IIT Research Institute ATTN: GACIAC 10 West 35th Street Chicago, Illinois 60616	1
Science Applications, Inc. 2109 West Clinton Ave. ATTN: Dr. W. Gene Cantrell Huntsville, AL 35805	1
DRCPM-PE-E, Mr. Noller	1
-PE-EA, Mr. Wagner	1
Mr. Bond	1
DRSMI-LP, Mr. Voigt	1
-R, Dr. McCorkle	1
-RG,	1
-RGL, Mr. White	5
DRSMI-RX	1

DISTRIBUTION (Concluded)

		No. Of Copies
DRSMI-RGG,	Mr. Ciliax	1
-RGN,	Mr. McLean	1
-RSL,	Mr. Bounds	1
-RBD		3
-RPT	(Record Set)	1
	(Reference Copy)	1
-RPR		15

END

DATE
FILMED
1-82

DTIC